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Least dissipation principle of heat transport potential capacity and its application in heat conduction optimization

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Abstract **In the viewpoint of heat transfer, heat transport potential capacity and its dissipation are defined based on the essence of heat transport phenomenon. Respectively, their physical meanings are the overall heat transfer capability and the dissipation rate of the heat transfer capacity. Then the least dissipation principle of heat transport potential capacity is presented to enhance the heat conduction efficiency in the heat conduction optimization. The principle is, for a conduction process with the constant integral of the thermal conductivity over the region, the optimal distribution of thermal conductivity, which corresponds to the highest heat conduction efficiency, is characterized by the least dissipation of heat transport potential capacity. Finally the principle is applied to some cases in heat conduction optimization.**

Keywords: heat transfer, heat transport potential capacity, heat conduction optimization, dissipation of heat transport potential capacity.

The heat conduction following the Fourier law widely exists in nature and engineering. Usually, the thermal resistance is applied to evaluating the performance of the heat conduction, i.e. the less resistance corresponds to the better performance. Therefore, the heat conduction is often enhanced by means of using high conductivity materials or reducing the thermal contact resistance. The more general performance criterion is the heat duty for the given temperature difference ΔT, or the temperature difference for the given heat duty. Although these criteria are obvious and suitable for one dimension problems, they are short of the generalization because the definition of temperature difference is of considerable arbitrariness in the multidimensional cases or the cases with complex boundary conditions. This arbitrariness makes it incapable to evaluate the heat conduction performance and consequently, optimize the heat conduction process.

In the viewpoint of thermodynamics, heat transfer is an irreversible process which belongs to the area of the nonequilibrium thermodynamics. Because of the successful application of variational principle in mechanics, Onsager[1,2] attempted to use variational principle to formulate the basic equations for nonequilibrium thermodynamics phenomena early in 1931, and the principle of the least energy dissipation was presented. Independent of the least energy dissipation principle, the principle of minimum entropy production was presented by Prigogine[3] in 1945 when he studied the thermal system in stationary nonequilibrium state. In 1955, Biot[4,5] developed a variational form of heat conduction equation based on the theory of irreversible thermodynamics. This equation is analogue to the principle of Hamilton in mechanics. Then Biot defined thermal potential and dissipation function of heat transfer, and established the Lagrangian form of the equations for the generalized coordinates. However, the physical meanings of these two definitions have not been further discussed. The Lagrangian equation is only used for the approximate solution of heat conduction with conductivity anisotropy. Yang[6] investigated the physical essence of the variational principle and functional analysis of the steady-state heat diffusion equation. When he discussed the heat conduction from the viewpoint of entropy production, he thought that the functional and its variational principles of the heat diffusion equation following the Fourier law have no physical meaning. Zeng[7] discussed systemically the variational principle of nonequilibrium thermodynamics, and introduced the application of the nonequilibrium thermodynamics in the transport processes in engineering, including the heat transport process with the outside electronic or magnetic fields acting on it. However, the discussion was focused on the entropy production with stationary nonequilibrium state being most stable. Bejan[8] developed the entropy production equation in the fluid flow and heat transfer, and optimized the geometrical parameters of the heat transfer devices in convection to minimize the entropy production, which is produced by the heat transfer and vicious dissipation.

1 Principle of the least energy dissipation and that of minimum entropy production[7]

For a long time, some physicists have attempted to describe the local entropy production by combining the second law of thermodynamics with the conservation equations of the mass, the momentum and the energy. It was not achieved until Onsager presented the Onsager reciprocity relations starting from the generality of the irreversible processes in 1931. All irreversible processes were considered as the consequences of the generalized thermodynamics fluxes produced by some generalized thermodynamics forces. The relations between forces and fluxes were assumed to be approximately linear. Afterwards, Meixner et al. established the expression of entropy production in some cases and consequently, created a new field — the irreversible thermodynamics or the nonequilibrium thermodynamics. It mainly studies universal manipulation of the phenomenological laws in the transport processes and the stationary state, the stability, the dynamics in the irreversible process. Appling the variational principle to thermodynamics, Onsager presented the principle of the least energy dissipation. At beginning the principle was given in the discussion of anisotropic heat conduction, afterwards, it was extended to the adiabatic isolated discontinuous system[1,2], while Prigogine[3] presented the principle of minimum entropy production independently. The difference between the two principles was not clarified until 1967.

The dissipation function is defined as

$$f = -\frac{1}{2} x_{\mathrm{D}} \cdot J_k, \tag{1}$$

where x_{D} is the dissipation force, which is proportional to the generalized flux J_k, that is

$$x_{\mathrm{D}} = -R_{ik} J_k, \tag{2}$$

where R_{ik} is a matrix of resistance coefficients. When the generalized forces and fluxes follow the linear phenomenological laws, the entropy production is

$$s = \sum_{i,k=1}^{f} L_{ik} X_i X_k \geq 0, \tag{3}$$

where L_{ik} is the phenomenological coefficient of the transport process. Hence, $f = \frac{1}{2} s$, i.e. the dissipation function is half of the entropy production. In the linear nonequilibrium thermodynamics the dissipation function can be directly expressed by means of forces or fluxes. By means of forces,

$$y(X, X) = \frac{1}{2} \sum_{i,k=1}^{f} L_{ik} X_i X_k \geq 0. \tag{4a}$$

By means of fluxes,

$$f(J, J) = \frac{1}{2} \sum_{i,k=1}^{f} R_{ik} J_i J_k \geq 0. \tag{4b}$$

The entropy representation should be used in the non-isothermal system. The energy representation is often used in isothermal system, i.e. the energy dissipation function Ts, $y^* = Ty$, $f^* = Tf$. Hence, the principle of the least energy dissipation and the principle of minimum entropy production are congruous in isothermal system.

The principle of minimum entropy production is stated that[3] when the system is described by linear phenomenological laws with constant coefficients satisfying the Onsager reciprocity relations, the stationary nonequilibrium states of the system are characterized by a minimum of the entropy production subject to the boundary conditions.

With the local entropy production expression, the correspondences can be proved between the stationary state and the state with the minimum entropy production. With the integral expression of entropy production, the heat diffusion equation, mass diffusion equation and N-S equation can be derived from the principle of minimum entropy production.

2 Least dissipation principle of heat transport potential capacity

(ⅰ) Heat transport potential capacity. The introduction in preceding section shows that the principle of the least energy dissipation and the principle of minimum entropy production are not relevant to the evaluation and optimization of the heat conduction performance.

In this report we focus on how to obtain approach and criterion, which can quantitatively evaluate the heat conduction performance and determine the heat transport efficiency. We begin with the physical conception. There are two preconditions for heat transport between objects. One is temperature difference. Therefore, the temperature is the potential for heat transport relative to the state of T = 0 K. But having the potential only is not enough for heat transport, the other precondition is that the objects must hold some heat capacity. So the product of the heat capacity of object and its temperature represents the object's overall capability for heat transport. Therefore, we define

$$Z_{\mathrm{tr}} = \frac{1}{2} QT = \frac{1}{2} M C_{\mathrm{p}} T^2 \tag{5}$$

as the heat transport potential capacity. Its dimension is J・K, where M is the object's mass, and C_{p} the specific heat. Biot defined the half of the product of the heat capacity and temperature as thermal potential and thought it similar to the potential (energy) of the system, but its physical meaning had not been further discussed. Here we indicate definitely that the temperature T is the potential of heat transport but not the potential of heat and work conversion. It is the potential capacity but not the potential that dissipates during the heat transport process, so the physical meaning of $Z_{\mathrm{tr}} = \frac{1}{2} QT$ is heat transport potential capacity, but not the thermal potential.

(ⅱ) Dissipation of heat transport potential capacity. Heat transport process is irreversible. When the heat Q is transferred from the high temperature object to the lower temperature object, the potential capacity must dissipate, i.e. the overall capability of heat transport must decrease. In heat conduction, the thermodynamic force is $X = -\nabla T$, the flux (heat flux) is $J = q = -k\nabla T$. Then the dissipation of heat transport potential capacity is half of the product of the force and flux, $y = \frac{1}{2} X \cdot J$, that is

$$y = \frac{1}{2} k (\nabla T)^2 = \frac{1}{2} \frac{q^2}{k}. \tag{6}$$

Its dimension is J・K/(m^3・s). The physical meaning is the dissipation (decrease) of heat transport potential capacity per unit volume and per unit time in the process of heat conduction. The dissipated is the potential capacity, so y is called the dissipation function of heat transport potential capacity. The dissipation of potential capacity during a heat transport process is equal to the integral of y over volume and time

$$Z_{\text{dis}} = \int_{W,\,t} y \,\mathrm{d}W\mathrm{d}t. \tag{7}$$

(**iii**) Heat transfer efficiency. As mentioned above, one object holds the heat transport potential capacity $Z_n = \frac{1}{2}QT$ with dimension J·K, and its physical meaning is the maximum possible potential capability to be transferred under infinitesimal temperature difference (for example, if the thermal conductivity $k =$), that is, there is no dissipation of the potential capacity in this case. Curve 1 in Fig. 1 shows such an ideal transient heat conduction process. In reality, the definite temperature difference always exists in heat transfer, so that the dissipation of heat transport potential capacity is inevitable. Therefore, more transport potential decreases than that in the ideal process for the given heat duty or less heat is transferred for the given temperature difference. Hence more potential capacity will dissipate because of the definite temperature difference. Curve 2 in Fig. 1 shows the change of potential capacity during the practical process, where Z_{dis} is the dissipation of potential capacity. At first increasing the heat transfer rate leads to the dissipation increment. As the normal state is reached, the decrease both in flux and the temperature gradient conduces to the gradual decrease of the heat transport potential capacity dissipation, and finally down to zero.

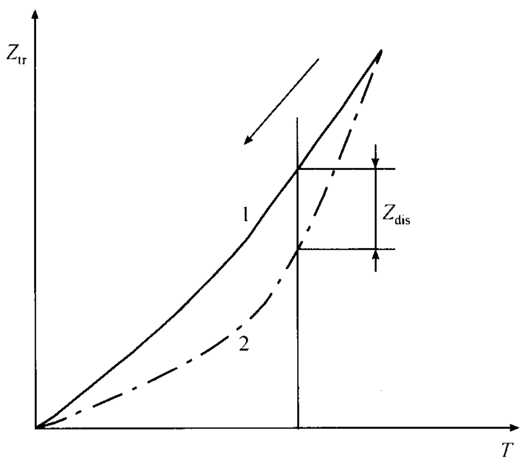


Fig. 1. Variation of heat transport potential capacity in heat conduction.

Therefore, the heat transfer efficiency can be defined as the ratio of output potential capacity to the input potential capacity (shown in Fig. 2).

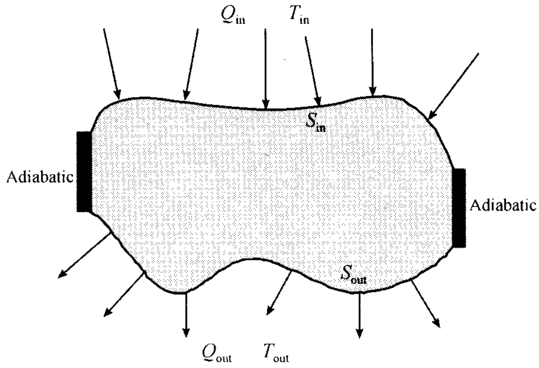


Fig. 2. Sketch of heat conduction in an object.

$$h = \frac{\sum Q_{\text{out}} T_{\text{out}}}{\sum Q_{\text{in}} T_{\text{in}}} = \frac{Z_{\text{tr,in}} - Z_{\text{dis}}}{Z_{\text{tr,in}}} = \frac{Z_{\text{tr,out}}}{Z_{\text{tr,in}}}. \tag{8}$$

The less dissipation of potential capacity implies the higher heat transfer efficiency, which, as a criterion for evaluating heat transfer performance, is generally applicable. In the one-dimension cases, this criterion may reduce the less temperature difference for the given heat duty or the more heat duty for the given temperature difference, which represent the higher heat transfer efficiency ç.

(**iv**) Principle of least potential capacity dissipation. Eq. (6) obviously shows that the dissipation of heat transport potential capacity decrease with the thermal conductivity increasing. Whereas, if there are different kinds of materials with different thermal conductivity in the region, is there any optimal distribution of the high conductivity materials which correspondents to the highest heat conduction efficiency? That is, the thermal captivity is a space function for the given overall heat conduction capability, which is the integral of the thermal conductivity over the region, i.e.

$$\int_W k\mathrm{d}W = \text{const.} \tag{9}$$

Different distributions of thermal conductivity lead to the different heat transfer rates and consequently, the different dissipation of heat transport potential capacity. Then the statement of the least dissipation principle of heat transport potential capacity is, "The heat conduction must make the heat transport potential capacity dissipate. When the integral of the thermal conductivity over the region is constant or the high conductivity materials are given, the optimal distribution of thermal conductivity, which is characterized by the least dissipation of heat transport potential capacity, leads to the highest heat conduction efficiency."

3 Application in heat conduction optimization

According to the least dissipation principle of the heat transport potential capacity, some heat conduction cases can be optimized to raise the heat conduction efficiency.

(i) One-dimension heat conduction in the cylinder. Consider an insulated cylinder layer out of the electric conducting wire shown in Fig. 3.

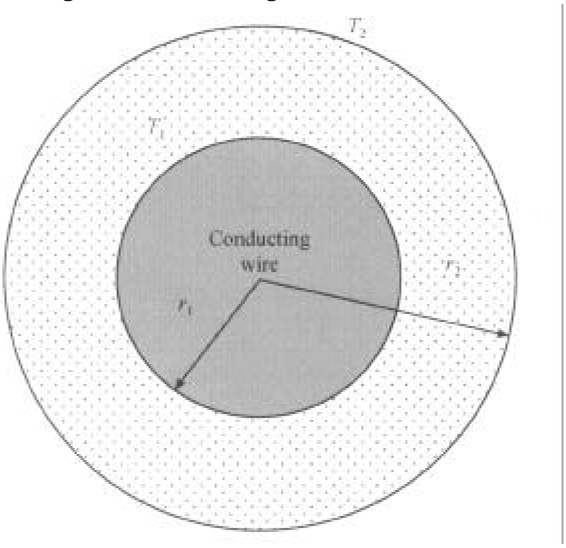


Fig. 3. Heat dissipation in the insulated layer.

The temperature difference is kept constant between the exterior surface and the interior surface, and the radial density of the insulated layer is variable under the condition of the constant gross weight, that is $\int r\, r\mathrm{d}r = \text{const}$. Because the thermal conductivity of most engineering materials is proportional to the density r, the above constraint condition may turn to $\int_{r_1}^{r_2} kr\mathrm{d}r \;\; = \text{const}$.

According to the least dissipation principle of heat transport potential capacity, the optimal radial distribution of density or thermal conductivity is characterized by least dissipation of heat transport potential capacity, that is,

$$d\,J = d\left(\int_{r_1}^{r_2} \frac{q^2}{k} r\mathrm{d}r\right) = 0. \tag{10}$$

This is an extreme value problem in functional analysis. let $y = \int_{r_1}^{r} kr\mathrm{d}r$, it follows that $y(r_1) = 0,\ y(r_2) = K_0$, then eq. (10) can be expressed as

$$d\,J(y) = d\left(\int_{r_1}^{r_2} \frac{(qr)^2}{y'} \mathrm{d}r\right) = 0. \tag{11}$$

The solution of eq. (11) is

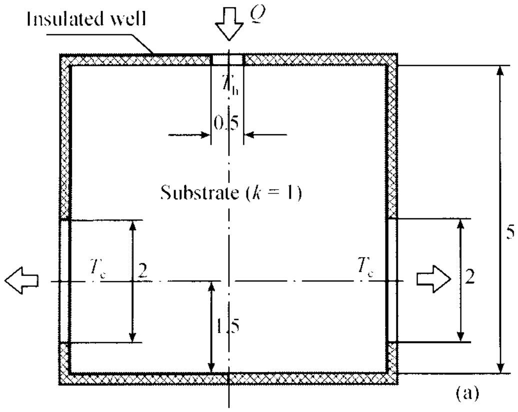
$$k = Cq, \textbf{ or } \frac{\mathrm{d}T}{\mathrm{d}r} = C, \tag{12}$$

where C is a constant. Solution (12) shows that the optimal distribution of thermal conductivity or the density, which corresponds to the maximum heat flux, is in direct proportion to the local heat flux, or in inverse proportion to the radius r. It can also be observed that the temperature gradient is uniform all over the region for the optimal distribution of the thermal conductivity.

(ii) The volume-to-point problem. The volume-to-point problem is how to effectively conduct the heat generated in a finite-size volume to a small patch (point), which is located on its boundary, with inserting high conductivity material[9]. When only finite amount of high conductivity materials are available, there exists an optimal construct of the materials. The problem can be abstracted as, "Consider one region with arbitrary shape and fixed heat duty which include the local heat source and the inflow heat from the outside. Determine the optimal distribution of thermal conductivity for the given integral of thermal conductivity over the region."

According to the least dissipation principle of heat transport potential capacity, the optimal distribution of thermal conductivity should be the same as that in the one-dimensional problem, i.e. the thermal conductivity is in direct proportion to the local heat flux.

In the two-dimension volume-to-point problem shown in Fig. 4(a), the heat comes from the patch at high temperature T_h, and goes out from the patch at low temperature T_c. It is assumed that the thermal conductivity of the substrate material is 1, and the conductivity of inserts is 10. The thermal conductivity is not continuous because there are two kinds of materials with different conductivities in the region. Then the optimal construct of the high conductivity materials can be obtained by the bionic optimization method[10]. In this method the high conductivity material is regarded as the alive. The physical conduction space and the physical feature, such as the geometric structure, the thermal boundary condition, the heat generation in the conduction domain, and the thermal conductivity of the substrate material are all regarded as the natural environment. The high conductivity material grows according to the evolutionary principle: grows towards the largest temperature gradient in the conduction region. As the criterion above mentioned, the more uniform the temperature gradient is, the less the dissipation of heat transport potential capacity decreases and the highest the heat conduction efficiency is. Since the high conductivity material at the location of the largest temperature gradient makes the temperature gradient field more uniform, the evolutionary principle conforms to the least dissipation principle of heat transport potential capacity. The numerical results show that the optimal construct of the high conductivity material is characterized by the least dissipation of heat transport potential capacity.



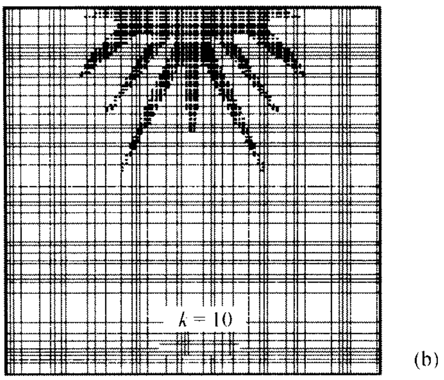


Fig. 4. (a) Two-dimension volume-to-point problem; (b) Shape of the high conductivity material.

4 Difference between the least dissipation principle of heat transport potential capacity and that of minimum entropy production

Entropy is the measure of the conversion extent from heat to work; and entropy production is the measure of the reduction of the doing work capability due to the irreversibility of the process. The principle of minimum entropy production indicates that the stationary nonequilibrium state is characterized by the minimum entropy production. All these concepts are discussed from the viewpoint of thermodynamics. However, what we are concerned with is the heat transport efficiency. The heat transport potential capacity denotes the overall capability of heat transport. The dissipation of the heat transport potential capacity represents the loss of the heat transport potential capacity. The principle of the least dissipation of transport potential capacity indicates that the optimal heat conduction process is characterized by the least dissipation of heat transport potential capacity.

In the case of volume-to-point problem, the numerical results show that the maximum heat transport rate with the object of the minimum entropy production is less than that with the object of the minimum dissipation of heat transport potential capacity for the optimal distribution of thermal conductivity.

5 Conclusions

Based on the essence of the heat transport, the heat transport potential capacity and its dissipation are defined from the angle of heat transfer. The corresponding physical meanings are the overall capability of heat transport and the loss of the heat transport potential capacity respectively. The entropy and entropy production in the existing literatures are all discussed in the viewpoint of thermodynamics.

The least dissipation principle of heat transport potential capacity is presented, and its statement is the heat conduction which must make the heat transport potential capacity dissipate. For a conduction process with the constant integral of the thermal conductivity over the region, the optimal distribution of thermal conductivity, which corresponds to the highest heat conduction efficiency, is characterized by the least dissipation of heat transport potential capacity. The principle can be applied to the optimization of heat conduction in order to enhance the heat conduction efficiency. However, the principle of the minimum entropy production is applicable for the irreversible process optimization to enhance the capability of doing work.

According to the least dissipation principle of heat transport potential capacity, the optimal distribution of thermal conductivity (high conductivity material) is characterized by the more uniform temperature gradient field while the heat conduction is optimum.

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